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Super Yangian Double and its Central Extension

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Abstract

We introduce super Yangian double $DY_\hbar[gl(m|n)]$ and its central extension $DY_\hbar[\widehat{gl(m|n)}]$. We give their defining relations in terms of current generators and obtain Drinfeld comultiplication.

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This paper concerns with Drinfeld current realization [1] of super Yangian double $DY_\hbar[gl(m|n)]$ and its central extension $DY_\hbar[\widehat{gl(m|n)}]$.

The Yangian double [2] $DY_\hbar(\mathcal{G})$ of a simple bosonic Lie algebra $\mathcal{G}$ is a quantum double of the Yangian $Y_\hbar(\mathcal{G})$ [1]. It is a deformation of the entire loop algebra and has important applications in massive integrable models [3, 4].

The Yangian double with center (or central extension of the Yangian double) $\widehat{DY_\hbar(\mathcal{G})}$ for $\mathcal{G} = gl(n),\ sl(n)$ were introduced in [5, 6] in terms of Drinfeld current generators.

The philosophy behind this paper is to introduce super Yangian double $DY_\hbar[gl(m|n)]$ and its central extension $DY_\hbar[\widehat{gl(m|n)}]$. This is achieved by generalizing the Reshetikhin and Semenov-Tian-Shansky (RS) construction [7] to the supersymmetric case. Using this super RS construction and Gauss decomposition [8], we obtain the defining relations for $DY_\hbar[\widehat{gl(m|n)}]$ in terms of super current generators. The computation in this paper is parallel to our recent work [9] on Drinfeld current realization of quantum affine superalgebra $U_q[gl(m|n)^{(1)}]$ (see also [10] for the special case of $m = n = 1$), which in some sense is a superization of work [11].

The graded Yang-Baxter equation (YBE) with spectral-parameter dependence takes the form

$$R_{12}(u-v)R_{13}(u)R_{23}(v) = R_{23}(v)R_{13}(u)R_{12}(u-v), \tag{1}$$

where $R(u) \in End(V \otimes V)$ with V being graded vector space and obeys the weight conservation condition: $R(u)_{\alpha\beta}^{\alpha'\beta'} \neq 0$ only when $[\alpha'] + [\beta'] + [\alpha] + [\beta] = 0$ mod2. The multiplication rule for the tensor product is defined for homogeneous elements $a,\ b,\ c,\ d$ of a quantum superalgebra by

$$(a \otimes b)(c \otimes d) = (-1)^{[b][c]}\,(ac \otimes bd) \tag{2}$$

where $[a] \in \mathbf{Z}_2$ denotes the grading of the element a.

Introduce the graded permutation operator P on the tensor product module $V \otimes V$ such that $P(v_\alpha \otimes v_\beta) = (-1)^{[\alpha][\beta]}(v_\beta \otimes v_\alpha)\,,\ \forall v_\alpha, v_\beta \in V$. In most cases R-matrix enjoys, among others, the following properties

$$(i)\ P_{12}R_{12}(u)P_{12} = R_{21}(u), \tag{3}$$

$$(ii)\ R_{12}(u)R_{21}(-u) = 1. \tag{4}$$

The graded YBE, when written in matrix form, carries extra signs [13, 12],

$$\begin{aligned} &R(u-v)_{\alpha\beta}^{\alpha'\beta'}\,R(u)_{\alpha'\gamma}^{\alpha''\gamma'}\,R(v)_{\beta'\gamma'}^{\beta''\gamma''}\,(-1)^{[\alpha][\beta]+[\gamma][\alpha']+[\gamma'][\beta']} \\ &\quad = R(v)_{\beta\gamma}^{\beta'\gamma'}\,R(u)_{\alpha\gamma'}^{\alpha'\gamma''}\,R(u-v)_{\alpha'\beta'}^{\alpha''\beta''}\,(-1)^{[\beta][\gamma]+[\gamma'][\alpha]+[\beta'][\alpha']}. \end{aligned} \tag{5}$$

In [9] we generalized the RS construction [7] to supersymmetric cases and obtained Drinfeld realization of the quantum affine superalgebra $U_q[gl(m|n)^{(1)}]$. Here we consider rational solution $R(u)$ to the graded YBE and give a 'rational' super RS algebra:

Definition 1 *: Rational super RS algebra is generated by invertible $L^{\pm}(u)$, satisfying*

$$\begin{aligned} R(u-v)L_1^{\pm}(u)L_2^{\pm}(v) &= L_2^{\pm}(v)L_1^{\pm}(u)R(u-v), \\ R(u_+-v_-)L_1^{+}(u)L_2^{-}(v) &= L_2^{-}(v)L_1^{+}(u)R(u_--v_+), \end{aligned} \tag{6}$$

where $L_1^{\pm}(u) = L^{\pm}(u) \otimes 1$, $L_2^{\pm}(u) = 1 \otimes L^{\pm}(u)$ and $u_{\pm} = u \pm \frac{1}{2}\hbar c$. For the first formula of (6), the expansion direction of $R(u-v)$ can be chosen in $\frac{u}{v}$ or $\frac{v}{u}$, but for the second formula, the expansion direction must only be in $\frac{u}{v}$.

In matrix form, (6) carries extra signs due to the graded multiplication rule of tensor products:

$$\begin{aligned} &R(u-v)_{\alpha\beta}^{\alpha"\beta"} L^{\pm}(u)_{\alpha"}^{\alpha'} L^{\pm}(v)_{\beta"}^{\beta'} (-1)^{[\alpha']([\beta']+[\beta"])} \\ &\qquad = L^{\pm}(v)_{\beta}^{\beta"} L^{\pm}(u)_{\alpha}^{\alpha"} R(u-v)_{\alpha"\beta"}^{\alpha'\beta'} (-1)^{[\alpha]([\beta]+[\beta"])}, \\ &R(u_+-v_-)_{\alpha\beta}^{\alpha"\beta"} L^{+}(u)_{\alpha"}^{\alpha'} L^{-}(v)_{\beta"}^{\beta'} (-1)^{[\alpha']([\beta']+[\beta"])} \\ &\qquad = L^{-}(v)_{\beta}^{\beta"} L^{+}(u)_{\alpha}^{\alpha"} R(u_--v_+)_{\alpha"\beta"}^{\alpha'\beta'} (-1)^{[\alpha]([\beta]+[\beta"])}. \end{aligned} \tag{7}$$

Introduce matrix θ:

$$\theta_{\alpha\beta}^{\alpha'\beta'} = (-1)^{[\alpha][\beta]}\delta_{\alpha\alpha'}\delta_{\beta\beta'}. \tag{8}$$

With the help of this matrix θ, one can cast (7) into the usual matrix equation,

$$\begin{aligned} R(u-v)L_1^{\pm}(u)\theta L_2^{\pm}(v)\theta &= \theta L_2^{\pm}(v)\theta L_1^{\pm}(u)R(u-v), \\ R(u_+-v_-)L_1^{+}(u)\theta L_2^{-}(v)\theta &= \theta L_2^{-}(v)\theta L_1^{+}(u)R(u_--v_+). \end{aligned} \tag{9}$$

Now the multiplications in (9) are simply the usual matrix multiplications.

We will take $R(u) \in End(V \otimes V)$ to be the Yang's rational R-matrix associated with superalgebra $gl(m|n)$,

$$R(u) = \frac{1}{u+2\hbar}(uI + 2\hbar P), \tag{10}$$

where V is a $(m+n)$-dimensional graded vector space. Let basis vectors $\{v_1,\ v_2, \cdots,\ v_m\}$ be even and $\{v_{m+1},\ v_{m+2}, \cdots,\ v_{m+n}\}$ be odd. Then the R-matrix has the following matrix elements:

$$\begin{aligned} R(u)_{\alpha\beta}^{\alpha'\beta'} &= (-1)^{[\alpha][\beta]}\tilde{R}(u)_{\alpha\beta}^{\alpha'\beta'}, \\ \tilde{R}(u) &= \sum_{i=1}^{m} E_i^i \otimes E_i^i + \sum_{i=m+1}^{m+n} \frac{2\hbar-u}{u+2\hbar} E_i^i \otimes E_i^i + \frac{u}{u+2\hbar}\sum_{i\neq j}(-1)^{[i][j]} E_i^i \otimes E_j^j \\ &\quad \sum_{i<j} \frac{2\hbar}{u+2\hbar} E_i^j \otimes E_j^i + \sum_{i>j} \frac{2\hbar}{u+2\hbar} E_i^j \otimes E_j^i. \end{aligned} \tag{11}$$

It is easy to check that the R-matrix $R(u)$ satisfies (3) and (4). We will construct central extended super Yangian double $DY_\hbar[\widehat{gl(m|n)}]$.

Theorem 1 *: $L^\pm(u)$ has the following Gauss decomposition*

$$
\begin{aligned}
L^\pm(u) \;=\; & \begin{pmatrix} 1 & \cdots & & 0 \\ e^\pm_{2,1}(u) & \ddots & & \\ e^\pm_{3,1}(u) & & & \vdots \\ \vdots & & & \\ e^\pm_{m+n,1}(u) & \cdots & e^\pm_{m+n,m+n-1}(u) & 1 \end{pmatrix} \begin{pmatrix} k^\pm_1(u) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k^\pm_{m+n}(u) \end{pmatrix} \\
& \times \begin{pmatrix} 1 & f^\pm_{1,2}(u) & f^\pm_{1,3}(u) & \cdots & f^\pm_{1,m+n}(u) \\ \vdots & \ddots & \cdots & & \vdots \\ & & & & f^\pm_{m+n-1,m+n}(u) \\ 0 & & & & 1 \end{pmatrix},
\end{aligned}
\tag{12}
$$

where $e^\pm_{i,j}(u)$, $f^\pm_{j,i}(u)$ and $k^\pm_i(u)$ $(i>j)$ are elements in the rational super RS algebra and $k^\pm_i(u)$ are invertible. Let

$$
\begin{aligned}
X^-_i(u) &= f^+_{i,i+1}(u_+) - f^-_{i,i+1}(u_-), \\
X^+_i(u) &= e^+_{i+1,i}(u_-) - e^-_{i+1,i}(u_+),
\end{aligned}
\tag{13}
$$

where $u_\pm = u \pm \frac{1}{2}\hbar c$, then c, $X^\pm_i(u)$, $k^\pm_j(u)$, $i = 1,2,\cdots,m+n-1$, $j = 1,2,\cdots,m+n$ give the defining relations of central extended super Yangian double $DY_\hbar[\widehat{gl(m|n)}]$ which, when $c=0$ reduce to those of super Yangian double $DY_\hbar[gl(m|n)]$.

The Gauss decomposition implies that the elements $e^\pm_{i,j}(u)$, $f^\pm_{j,i}(u)$ $(i>j)$ and $k^\pm_i(u)$ are uniquely determined by $L^\pm(u)$. In the following we will denote $f^\pm_{i,i+1}(u)$, $e^\pm_{i+1,i}(u)$ as $f^\pm_i(u)$, $e^\pm_i(u)$, respectively.

The following matrix equations can be deduced from (9):

$$
R_{21}(u-v)\theta L^\pm_2(u)\theta L^\pm_1(v) = L^\pm_1(v)\theta L^\pm_2(u)\theta R_{21}(u-v), \tag{14}
$$

$$
R_{21}(u_- - v_+)\theta L^-_2(u)\theta L^+_1(v) = L^+_1(v)\theta L^-_2(u)\theta R_{21}(u_+ - v_-), \tag{15}
$$

$$
\theta L^\pm_2(u)^{-1}\theta L^\pm_1(v)^{-1}R_{21}(u-v) = R_{21}(u-v)L^\pm_1(v)^{-1}\theta L^\pm_2(u)^{-1}\theta, \tag{16}
$$

$$
\theta L^+_2(u)^{-1}\theta L^-_1(v)^{-1}R_{21}(u_+ - v_-) = R_{21}(u_- - v_+)L^-_1(v)^{-1}\theta L^+_2(u)^{-1}\theta, \tag{17}
$$

$$
L^\pm_1(v)^{-1}R_{21}(u-v)\theta L^\pm_2(u)\theta = \theta L^\pm_2(u)\theta R_{21}(u-v)L^\pm_1(v)^{-1}, \tag{18}
$$

$$
L^-_1(v)^{-1}R_{21}(u_+ - v_-)\theta L^+_2(u)\theta = \theta L^+_2(u)\theta R_{21}(u_- - v_+)L^-_1(v)^{-1}, \tag{19}
$$

$$
L^+_1(v)^{-1}R_{21}(u_- - v_+)\theta L^-_2(u)\theta = \theta L^-_2(u)\theta R_{21}(u_+ - v_-)L^+_1(v)^{-1}, \tag{20}
$$

where $R_{21}(u-v) = R(v-u)^{-1}$. As in (9), the multiplications in (14 – 20) are usual matrix multiplications.

Using (8), (11), (9, 14 – 20) and theorem 1, and by parallel calculations as to [9], we obtain

Definition 2 *: $DY_\hbar[\widehat{gl(m|n)}]$ is an associative algebra over the ring of formal power series in the variable $\hbar$ and with Drinfeld current generators: $X_i^\pm(u),\ k_j^\pm(u),\ i = 1, 2, \cdots, m+n-1,\ j = 1, 2, \cdots, m+n$ and a central element c. $k_i^\pm(u)$ are invertible. The grading of the generators are: $[X_m^\pm(u)] = 1$ and zero otherwise. When $c = 0$, $DY_\hbar[\widehat{gl(m|n)}]$ reduces to $DY_\hbar[gl(m|n)]$. The defining relations are given by*

$$
\begin{aligned}
k_i^\pm(u)k_j^\pm(v) &= k_j^\pm(v)k_i^\pm(u), \quad i \neq j \\
k_i^+(u)k_i^-(v) &= k_i^-(v)k_i^+(u), \quad i \leq m, \\
\frac{u_+ - v_- - 2\hbar}{u_+ - v_- + 2\hbar}k_i^+(u)k_i^-(v) &= \frac{u_- - v_+ - 2\hbar}{u_- - v_+ + 2\hbar}k_i^-(v)k_i^+(u), \quad m < i \leq m+n, \\
\frac{u_\pm - v_\mp}{u_\pm - v_\mp + 2\hbar}k_i^\mp(v)^{-1}k_j^\pm(u) &= \frac{u_\mp - v_\pm}{u_\mp - v_\pm + 2\hbar}k_j^\pm(u)k_i^\mp(v)^{-1}, \quad i > j, \\
k_j^\pm(u)^{-1}X_i^-(v)k_j^\pm(u) &= X_i^-(v), \quad j - i \leq -1, \\
k_j^\pm(u)^{-1}X_i^+(v)k_j^\pm(u) &= X_i^+(v), \quad j - i \leq -1, \quad \text{or} \\
k_j^\pm(u)^{-1}X_i^-(v)k_j^\pm(u) &= X_i^-(v), \quad j - i \geq 2, \\
k_j^\pm(u)^{-1}X_i^+(v)k_j^\pm(u) &= X_i^+(v), \quad j - i \geq 2, \\
k_i^\pm(u)^{-1}X_i^-(v)k_i^\pm(u) &= \frac{u_\mp - v + 2\hbar}{u_\mp - v}X_i^-(v), \quad i < m, \\
k_i^\pm(u)^{-1}X_i^-(v)k_i^\pm(u) &= \frac{u_\mp - v - 2\hbar}{u_\mp - v}X_i^-(v), \quad m < i \leq m+n-1, \\
k_{i+1}^\pm(u)^{-1}X_i^-(v)k_{i+1}^\pm(u) &= \frac{u_\mp - v - 2\hbar}{u_\mp - v}X_i^-(v), \quad i < m, \\
k_{i+1}^\pm(u)^{-1}X_i^-(v)k_{i+1}^\pm(u) &= \frac{u_\mp - v + 2\hbar}{u_\mp - v}X_i^-(v), \quad m < i \leq m+n-1, \\
k_i^\pm(u)X_i^+(v)k_i^\pm(u)^{-1} &= \frac{u_\pm - v + 2\hbar}{u_\pm - v}X_i^+(v), \quad i < m, \\
k_i^\pm(u)X_i^+(v)k_i^\pm(u)^{-1} &= \frac{u_\pm - v - 2\hbar}{u_\pm - v}X_i^+(v), \quad m < i \leq m+n-1, \\
k_{i+1}^\pm(u)X_i^+(v)k_{i+1}^\pm(u)^{-1} &= \frac{u_\pm - v - 2\hbar}{u_\pm - v}X_i^+(v), \quad i < m, \\
k_{i+1}^\pm(u)X_i^+(v)k_{i+1}^\pm(u)^{-1} &= \frac{u_\pm - v + 2\hbar}{u_\pm - v}X_i^+(v), \quad m < i \leq m+n-1, \\
k_i^\pm(u)^{-1}X_m^-(v)k_i^\pm(u) &= \frac{u_\mp - v + 2\hbar}{u_\mp - v}X_m^-(v), \quad i = m,\ m+1, \\
k_i^\pm(u)X_m^+(v)k_i^\pm(u)^{-1} &= \frac{u_\pm - v + 2\hbar}{u_\pm - v}X_m^+(v), \quad i = m,\ m+1,
\end{aligned}
$$

$$
\begin{aligned}
(u-v\mp 2\hbar)X_i^{\mp}(u)X_i^{\mp}(v) &= (u-v\pm 2\hbar)X_i^{\mp}(v)X_i^{\mp}(u), \quad i<m,\\
(u-v\pm 2\hbar)X_i^{\mp}(u)X_i^{\mp}(v) &= (u-v\mp 2\hbar)X_i^{\mp}(v)X_i^{\mp}(u), \quad m<i\leq m+n-1,\\
\{X_m^{\pm}(u),X_m^{\pm}(v)\} &= 0,\\
(u-v)X_i^{+}(u)X_{i+1}^{+}(v) &= (u-v+2\hbar)X_{i+1}^{+}(v)X_i^{+}(u), \quad i<m,\\
(u-v)X_i^{+}(u)X_{i+1}^{+}(v) &= (u-v-2\hbar)X_{i+1}^{+}(v)X_i^{+}(u), \quad m\leq i\leq m+n-1,\\
(u-v+2\hbar)X_i^{-}(u)X_{i+1}^{-}(v) &= (u-v)X_{i+1}^{-}(v)X_i^{-}(u), \quad i<m,\\
(u-v-2\hbar)X_i^{-}(u)X_{i+1}^{-}(v) &= (u-v)X_{i+1}^{-}(v)X_i^{-}(u), \quad m\leq i\leq m+n-1,\\
[X_i^{+}(u),X_j^{-}(v)] &= -2\hbar\delta_{ij}\Big(\delta(u_- - v_+)k_{i+1}^{+}(v_+)k_i^{+}(v_+)^{-1}\\
&\qquad -\delta(u_+ - v_-)k_{i+1}^{-}(u_+)k_i^{-}(u_+)^{-1}\Big), \quad i,j\neq m,\\
\{X_m^{+}(u),X_m^{-}(v)\} &= 2\hbar\Big(\delta(u_- - v_+)k_{m+1}^{+}(v_+)k_m^{+}(v_+)^{-1}\\
&\qquad -\delta(u_+ - v_-)k_{m+1}^{-}(u_+)k_m^{-}(u_+)^{-1}\Big),
\end{aligned}
\tag{21}
$$

where $[X,Y]\equiv XY-YX$ stands for a commutator and $\{X,Y\}\equiv XY+YX$ for an anti-commutator and

$$
\delta(u-v)=\sum_{k\in\mathbf{Z}}u^k v^{-k-1} \tag{22}
$$

is a formal series, together with the following Serre and extra Serre [14, 15] relations:

$$
\begin{aligned}
&\{X_i^{\pm}(u_1)X_i^{\pm}(u_2)X_{i+1}^{\pm}(v)-2X_i^{\pm}(u_1)X_{i+1}^{\pm}(v)X_i^{\pm}(u_2)\\
&\qquad + X_{i+1}^{\pm}(v)X_i^{\pm}(u_1)X_i^{\pm}(u_2)\}+\{u_1\leftrightarrow u_2\}=0, \quad i\neq m,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&\{X_{i+1}^{\pm}(u_1)X_{i+1}^{\pm}(u_2)X_i^{\pm}(v)-2X_{i+1}^{\pm}(u_1)X_i^{\pm}(v)X_{i+1}^{\pm}(u_2)\\
&\qquad + X_i^{\pm}(v)X_{i+1}^{\pm}(u_1)X_{i+1}^{\pm}(u_2)\}+\{u_1\leftrightarrow u_2\}=0, \quad i\neq m-1,
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&\{(u_1-u_2\mp 2\hbar)[X_m^{\pm}(u_1)X_m^{\pm}(u_2)X_{m-1}^{\pm}(v)-2X_m^{\pm}(u_1)X_{m-1}^{\pm}(v)X_m^{\pm}(u_2)\\
&\qquad + X_{m-1}^{\pm}(v)X_m^{\pm}(u_1)X_m^{\pm}(u_2)]\}+\{u_1\leftrightarrow u_2\}=0,
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&\{(u_2-u_1\mp 2\hbar)[X_m^{\pm}(u_1)X_m^{\pm}(u_2)X_{m+1}^{\pm}(v)-2X_m^{\pm}(u_1)X_{m+1}^{\pm}(v)X_m^{\pm}(u_2)\\
&\qquad + X_{m+1}^{\pm}(v)X_m^{\pm}(u_1)X_m^{\pm}(u_2)]\}+\{u_1\leftrightarrow u_2\}=0,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\{(u_1-u_2\mp 2\hbar)[X_m^{\pm}(u_1)X_m^{\pm}(u_2)X_{m-1}^{\pm}(v_1)X_{m+1}^{\pm}(v_2)\\
&\qquad -2X_m^{\pm}(u_1)X_{m-1}^{\pm}(v_1)X_m^{\pm}(u_2)X_{m+1}^{\pm}(v_2)]\\
&\qquad \mp 4\hbar X_{m-1}^{\pm}(v_1)X_m^{\pm}(u_1)X_m^{\pm}(u_2)X_{m+1}^{\pm}(v_2)\\
&\qquad +(u_2-u_1\mp 2\hbar)[-2X_{m-1}^{\pm}(v_1)X_m^{\pm}(u_1)X_{m+1}^{\pm}(v_2)X_m^{\pm}(u_2)\\
&\qquad + X_{m-1}^{\pm}(v_1)X_{m+1}^{\pm}(v_2)X_m^{\pm}(u_1)X_m^{\pm}(u_2)]\}+\{u_1\leftrightarrow u_2\}=0.
\end{aligned}
\tag{27}
$$

Remark: For the special case of $m=n=1$, we have

$$
k_i^{\pm}(u)k_j^{\pm}(v) \;=\; k_j^{\pm}(v)k_i^{\pm}(u), \quad i,\ j=1,2,
$$

$$
\begin{aligned}
k_1^+(u)k_1^-(v) &= k_1^-(v)k_1^+(u),\\
\frac{u_+-v_--2\hbar}{u_+-v_-+2\hbar}k_2^+(u)k_2^-(v) &= \frac{u_--v_+-2\hbar}{u_--v_++2\hbar}k_2^-(v)k_2^+(u),\\
\frac{u_\pm-v_\mp}{u_\pm-v_\mp+2\hbar}k_2^\mp(v)^{-1}k_1^\pm(u) &= \frac{u_\mp-v_\pm}{u_\mp-v_\pm+2\hbar}k_1^\pm(u)k_2^\mp(v)^{-1},\\
k_i^\pm(u)^{-1}X_1^-(v)k_i^\pm(u) &= \frac{u_\mp-v+2\hbar}{u_\mp-v}X_1^-(v),\\
k_i^\pm(u)X_1^+(v)k_i^\pm(u)^{-1} &= \frac{u_\pm-v+2\hbar}{u_\pm-v}X_1^+(v),\\
\{X_1^\pm(u),X_1^\pm(v)\} &= 0,\\
\{X_1^+(u),X_1^-(v)\} &= 2\hbar\Big(\delta(u_--v_+)k_2^+(v_+)k_1^+(v_+)^{-1}\\
&\qquad -\delta(u_+-v_-)k_2^-(u_+)k_1^-(u_+)^{-1}\Big). \qquad (28)
\end{aligned}
$$

This is the defining relations of $DY_\hbar[\widehat{gl}(1|1)]$, which, when $c=0$, reduce to those of centraless super Yangian double $DY_\hbar[gl(1|1)]$ obtained in [16].

Theorem 2 *: The algebra $DY_\hbar[\widehat{gl}(m|n)]$ given by definition 2 has a Hopf algebra structure, which is given by the following formulae.*

Coproduct Δ

$$
\begin{aligned}
&\Delta(c)=c\otimes 1+1\otimes c,\\
&\Delta(k_j^\pm(u))=k_j^\pm(u\pm\frac{1}{2}\hbar c_2)\otimes k_j^\pm(u\mp\frac{1}{2}\hbar c_1),\quad j=1,2,\cdots,m+n,\\
&\Delta(X_i^+(u))=X_i^+(u)\otimes 1+\psi_i(u+\frac{1}{2}\hbar c_1)\otimes X_i^+(u+\hbar c_1),\\
&\Delta(X_i^-(u))=1\otimes X_i^-(u)+X_i^-(u+\hbar c_2)\otimes\phi_i(u+\frac{1}{2}\hbar c_2),\quad i=1,2,\cdots,m+n-1,
\end{aligned} \tag{29}
$$

where $c_1=c\otimes 1,\ c_2=1\otimes c,\ \psi_i(u)=k_{i+1}^-(u)k_i^-(u)^{-1}$ *and* $\phi_i(u)=k_{i+1}^+(u)k_i^+(u)^{-1}$.

Counit ϵ

$$
\epsilon(c)=0,\qquad \epsilon(k_j^\pm(u))=1,\qquad \epsilon(X_i^\pm(u))=0. \tag{30}
$$

Antipode S

$$
\begin{aligned}
&S(c)=-c,\qquad S(k_j^\pm(u))=k_j^\pm(u)^{-1},\\
&S(X_i^+(u))=-\psi_i(u-\frac{1}{2}\hbar c)^{-1}X_i^+(u-\hbar c),\\
&S(X_i^-(u))=-X_i^-(u-\hbar c)\phi_i(u-\frac{1}{2}\hbar c)^{-1}.
\end{aligned} \tag{31}
$$

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